'Obligatory' Implies 'Permitted'. Material or Formal Entailment?

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Abstract: *The paper discusses the possible logical status of the 'Obligatory implies Permitted' principle. It does so, by reconstructing the implication between 'obligatory' and 'permitted' first as a material entailment, depending on the world, and then as a formal entailment, depending on the logical forms of sentences. The main result of this inquiry is that, on the first material interpretation, the entailment is not granted universally, since it is only valid regarding consistent normative systems, and, on the second formal construction, the entailment is vacuous, since it is just a way of framing the relations of the concepts of 'obligation' and 'permission'.*

0. Basic Deontic Logic

It is a widespread view, in deontic logic and legal theory alike, that an obligation bearing upon a certain norm-content *p* entails a permission bearing on the same content *p*: this is, in short, the "Ought implies May" or "Obligatory implies Permitted" (OPP) principle. However, most deontic logicians and legal theorists do not seem to be fully aware of the consequences that committing to such an intuitive principle brings about.

In most systems of deontic logic OPP is a basic axiom, so that it has the nature of a stipulation[1]. In what follows, I shall try to demonstrate the "tricky" nature of such a principle. Before doing so, a caveat is in order: I shall assume that norms have logical values, though without committing to a particular logical value. I shall say, very generally and neutrally, that certain norms either hold (and hence have *1* as their logical value) or do not hold (and have *0* as their logical value).

Having said that, let us take the following traditional axioms to be the kernel of our system of deontic logic[2]:

(A_0) $Pp \leftrightarrow \sim O\sim p$
(A_1) $O(p \,\&\, q) \leftrightarrow (Op \,\&\, Oq)$
(OPP) $Op \rightarrow Pp$

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I would like to thank Riccardo Guastini, Francesca Poggi, and Jorge Rodríguez for helpful comments on a previous draft of the paper.

[1] Indeed, in the standard system of deontic logic, OPP is *the* principle which *formally* makes it possible to create deontic logic out of basic modal logic. See Palladino-Palladino (2007: 61).

[2] von Wright (1951). Axiom A_1 is far from being uncontroversial. Here, I mention it just to represent the traditional import of standard deontic logic, without necessarily upholding it. At any rate, in the pages that follow, it hardly plays any role.

From these axioms, and in particular from the inter-definability of "Permitted" and "Obligatory", we can derive both the principle of deontic consistency and the principle of deontic completeness. They are easily demonstrated[3].

Deontic Consistency

[1] $Pp \rightarrow \sim O \sim p$	left to right implication from A_0.
[2] $O \sim p \rightarrow \sim Pp$	contraposition in [1]
[3] $\sim(O \sim p \ \& \ Pp)$	definition of the conditional in [2]

Deontic Completeness

[4] $\sim O \sim p \rightarrow Pp$	right to left implication from A_0
[5] $O \sim p \vee Pp$	definition of the conditional in [4]

Sentence [3] provides us with the tenet that the norms "Forbidden p" and "Permitted p" cannot hold at the same time. In other words, normative systems are consistent.

In turn, sentence [5] provides us with the tenet that one of the norms "Forbidden p" or "Permitted p" holds. In other words, normative systems are complete[4].

1. Two Kinds of Entailment

In logic, two kinds of entailment are found: material and formal[5].

Material entailment ($\Rightarrow$) is the relation between two sentences "p" and "q" such that the resulting complex conditional sentence "$p \Rightarrow q$" is false if and only if "p" is true and "q" is false. Material entailment is verified or falsified depending on the actual truth-values of the atomic propositions which the conditional is made of.

By contrast, formal entailment ($\supset$) is a tautology. Hence, it is always true: it is, as it were, a "necessary" entailment. An example of such an entailment is the following: "$p \supset p \vee q$". There's no way that such an entailment turns out to be false, regardless of the truth values of the atomic sentences which compose it. Other famous examples are the following: "$p \ \& \ q \supset p$", or "$(p \supset q \ \& \sim q) \supset \sim p$". The truth-values of such an entailment depend on the logical form of the sentences which compose the conditional, not on the world.

[3] Here I follow closely Rodríguez (2003: 95-96).
[4] von Wright (1999: 32).
[5] Quine (1950: 14-18, 27-29).

When assessing the truth of the OPP, the most intriguing literature have dwelled on the ambiguity of deontic sentences, which are capable of being understood either as genuine norms or as meta-linguistic propositions bearing upon normative systems (in short, as normative propositions)[6]. In such a line of inquiry, the discovery of two different situations of "permission" in the logic of normative propositions has made it possible to build a logical account of actual normative systems.

This is, without a doubt, a correct and fruitful line of inquiry, whose importance must be born in mind. However, my point here will be that, even if we stick exclusively to a *prescriptive* interpretation of deontic sentences (i.e. as expressing norms), several problems arise, due to the unclear nature of the implication between the obligatory and the permitted.

2. OPP as a Material Entailment (or Material OPP)

If one reconstructs OPP as a material entailment (i.e. Op ⇒ Pp), one commits oneself to accepting that such a conditional does not hold whenever "Op" has the logical value *1* and "Pp" has the logical value *0*, as is shown by the following table.

	Op	Pp	~Op	~Pp	Op ⇒ Pp
1.	*1*	*1*	*0*	*0*	*1*
2.	*1*	*0*	*0*	*1*	*0*
3.	*0*	*1*	*1*	*0*	*1*
4.	*0*	*0*	*1*	*1*	*1*

Material OPP may be rephrased as follows:

[6] ~(Op & ~Pp)

This means that material OPP does not hold in case 2, where the normative system at hand is indeed formed by norms "Op" and "~Pp" (which is the norm-negation of Pp and is equivalent to "O~p"). In this very case, the system at hand is inconsistent, since "Op" and "O~p" are clearly incompatible. So, OPP holds *only* in relation to consistent normative systems.

Notice, moreover, that material OPP is validated in cases 3 and 4, where "Op" does not hold, and "Facultative or Optional p" (case 3) and "Forbidden p" (case 4) are respectively represented. This is to say, once more, that the principle

[6] Alchourrón (1969), Poggi (2000), Rodríguez (2003).

at hand only holds for consistently qualified states of affairs, whatever is their (maximal) qualification[7].

In addition to this, we can notice that OPP also requires that the normative system be complete. In fact, OPP is equivalent, because of the definition of conditional, to:

[7] ~Op ∨ Pp

From [7], because of the inter-definability of "Obligatory" and "Permitted", we can derive the tenet that at least one between the permission to *p* and the permission not to *p* holds in the system at hand. This is formalized as follows[8]:

[8] Pp ∨ P~p

This is plausible, obviously, only for *complete* systems, since in incomplete systems it may be the case that both *p* and its complementary *~p* are unqualified: indeed, this is the usual definition of a "normative gap".

As a consequence, material OPP only holds for complete and consistent systems. However, in reality, many (if not most) normative systems are not gapless and contradiction-free. If this is correct, we may affirm that, contrary to our intuitions, OPP has a quite marginal role to play in our logical reconstruction of actual normative systems, if we understand it as a material conditional.

3. OPP as a Formal Entailment (or Formal OPP)

What if we interpret OPP as a *formal* entailment, i.e. one that cannot be falsified by looking at the world?

The first, Quinean, interpretation is that the formal OPP has to do with concepts, not actual state of affairs. We all agree that if, following the mainstream in deontic logic, we build up the concept of "obligatory" as meaning "permitted to carry out and not permitted to refrain from carrying out", the concept of obligatory entails that of permitted. This of course is not informative[9]. However, theorists tend to construe this first reading as a tenet that tells us that whenever a certain normative authority enacts "Op", then she implicitly enacts also "Pp", despite possible inconsistencies in the actual normative system at hand. But this is just a mere stipulating move about what

[7] By "maximal qualifications" I mean those operators which qualify both a certain normative content and its complementary content. They are, accordingly, "obligatory", "forbidden", and "facultative (or optional)". Cf. Alchourrón-Bulygin (1971: 36 ff.).

[8] For the sake of simplicity, I have omitted the passage regarding the commutability of disjoints, according to which "P~p ∨ Pp" is equivalent to "Pp ∨ P~p".

[9] At least, not informative in the "purest sense" of providing pieces of information about the world. Of course, it tells us something (and is in this sense informative) about concepts.

we want to (or we should) infer from certain norms within a certain set of norms. In other words, it is either a vacuous tool[10] or a move which gets us back to the previous section on the problems of the material interpretation of OPP.

Another possible, and more interesting, interpretation consists in understanding formal OPP as a recommendation for the norm-giver[11], according to which the norm-giver *should* enact "Pp" if she enacted "Op". This is, obviously, a call for completeness of the formulation of a normative system[12], since the norm-giver is required to logically develop, by enacting implied rules, the norms she had previously enacted.

Another possible interpretation is that the norm-giver *should* not enact "~Pp" if she enacted "Op". This is a call for consistency of the normative system, derivable from a formal reading of [6]. However, we must be aware of the fact that, by doing so, we face the ambiguity of "permission" already within the scope of prescriptive reading of deontic sentences; whereas it is commonly held that it is a specific feature of the descriptive reading of such sentences.

In fact, the norm-giver could match the requirement at hand in two ways: either by enacting "Pp" or, more simply, by not enacting "~Pp". In the latter event, we would face a gap, as it were, regarding the permission of *p*. This gap, of course, might be filled up by enacting "Pp", but it must be highlighted that this option comes from complying with the *first* interpretation of the formal OPP as a recommendation to the norm-giver. It does not have anything to do, per se, with the *second* interpretation of the formal OPP. A perfectly "rational" norm-giver should conform to both principles at the same time.

We must finally notice that these ideas are not easy to apply to both moral and legal systems, as we normally conceive them.

Regarding the latter systems, the mentioned tenets imply that a lawgiver who intends to be complete must enact all the norms implied (on the basis of certain rules of inference) by enacted norms. But this also means that she is not allowed to enact norms that conflict with previously enacted norms. As everybody knows, this is an intuition that runs counter to the characteristically dynamic nature of legal systems.

These tenets do not fare better with moral systems either. Since moral systems typically have no norm-giver, it is not clear who should proceed to infer specific norms from "axiomatic" ones. To the extent that formal OPP is interpreted as a normative ideal addressed to a norm-giver and moral systems lack a norm-giver, this rule seems to be pointless[13].

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[10] "Vacuous" in the sense of "being opposite to a logically material issue".

[11] Von Wright (1999: 33-34). To be sure, such a reinterpretation should be better rephrased formally as "$O(Op \rightarrow Pp)$".

[12] Indeed, the normative system at hand would be *logically* complete, but incomplete regarding *the making explicit* of "relevant" derived norms.

[13] Of course, OPP here might be understood as a normative ideal of rational adequacy for normative systems lacking an authority. But, in my view, this would be paramount to surreptitiously state that either there is indeed a supra-sensorial authority or that interpreters, in the end, play the role of norm-givers.

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